Geomagnetic Attitude Control of an Axisymmetric Spinning Satellite

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The results of this study yield a general control law for geomagnetic attitude stabilization of a satellite. The proposed law, called the switching function, is derived from the asymptotic stability condition. Use of this law enables us to select the pattern of the onboard dipole moment satisfying the criteria for achieving the maximum effective torque and minimum transverse torque at every instant of activation. Analysis and formulation have been made for a simplified case and then extended to the general case including the canted geomagnetic dipole and the elliptical orbit. The derived switching point at which the polarity of a dipole is reversed is found to be different from those presented by the averaging method. This difference makes the spin axis attain the desired direction faster. The narrow pulse pattern activated halfway between the switching points is found advisable, in a practical application, since a considerably effective utilization of the applied torque is made possible. For spin rate control, operation around the selected point on the orbit is concluded desirable, satisfying the above criteria. Simulation runs have verified the feasibility of the law thus derived and showed a notable improvement in performance.

Nomenclature

$\mathbf{B}$	= geomagnetic induction vector
D_m	= geomagnetic dipole moment Me/R^3, $Me = 8.1 \times 10^{25}$ gauss-cm^3
$\mathbf{E}$	= error vector $(\mathbf{H}_f - \mathbf{H})$, $E \equiv \|\mathbf{E}\|$
e	= orbital eccentricity
$\mathbf{H}$	= angular momentum of the satellite
I_s, I_p	= moment of inertia around the spin axis and the transverse axes, respectively
$\mathbf{M}$	= applied magnetic dipole moment vector
m	= Earth's rotation rate
R	= radial distance from Earth's center
S	= switching function defined by Eq. (8) or (11)
$\mathbf{T}$	= magnetic control torque vector $(\mathbf{M} \times \mathbf{B})$
T_{eff}	= effective torque to reduce the error
$T_\perp$	= torque perpendicular to the effective torque
U, V	= respective magnitude of applied magnetic dipole moment ($\mathbf{M} = U\mathbf{k}_B$ for spin axis control and $\mathbf{M} = V\mathbf{i}_B$ for spin rate control)
α^2, β^2	= level of the dipole moment for spin axis control and spin rate control, respectively
γ	= longitude of the plane, containing both geographic and geomagnetic north pole, from the vernal equinox (Fig. 1)
δ	= geomagnetic polar axis declination, 11.4°
ζ	= anomaly angle of the orbital plane from the vernal equinox
η	= orbital angle from the ascending node
ψ, θ, ϕ	= azimuth and elevation angle of the spin axis, and spin angle, respectively, as defined in Fig. 2
θ_i	= orbital inclination
ξ	= Earth's rotation angle $(\gamma - \zeta)$
Φ_I, Φ_O, Φ_B	= inertial, orbital and body coordinate frame, respectively, as defined in Figs. 1 and 2
Ω	= spin rate
$\boldsymbol{\omega}$	= angular rate vector of the satellite
ω_p	= argument of perigee

Subscripts

I, O, B	= inertial, orbital, and body frame, respectively
f	= desired state
1, 2, 3	= refer to the i, j, k axes

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Introduction

USE of the geomagnetic field gives one of the simplest and most practical means of satellite attitude control, where the interaction between an onboard electromagnetic dipole moment and the geomagnetic field provides the control torque. A major problem involved in this type of attitude control is determining how one can generate the control law to govern the dipole for the desired control.

In previous investigations on the subject, there were roughly two approaches. The first approach, designated as the averaging method, was taken by Renard[1] and Grasshoff.[2] The torque produced was averaged, assuming the control pattern and the spin axis being sensibly invariant, over one interval of operation. The control law was chosen such that the averaged torque would produce the desired orientation. Renard[1] made a comparative study of control possibilities by five types of control pattern and concluded that the quarter orbit bang-bang pattern was especially suitable for near-polar orbits. Application of the quarter orbit pattern to the TIROS wheel satellite was described by Lindorfer and Muhefelder.[3] Another approach was taken by Wheeler,[4] Ergin and Wheeler,[5] Mobley,[6] and Sorensen.[8] Wheeler developed a law for circular orbits, specifying the error function consisting of attitude and attitude rate. Ergin and Wheeler[5] suggested the criteria of maximizing the scalar product of the magnetic torque and the error angular momentum. But their analysis was restricted to the wheel-type stabilization. The single pulse method, in which the control dipole was applied only when the most rapid desired orientation could be made, was presented by Mobley[6] and applied to the AE-B satellite. The applied dipole only took positive polarity. Sorensen[8] applied the Kalman filter technique to the estimation of the attitude error and developed the minimum energy control law using these estimates.

In this paper, a fairly general approach is taken by applying the switching function in such a manner that control is possible under any initial and desired condition and under any orbital condition. By analyzing the switching function, a general law both for the spin axis orientation and spin rate control is investigated, including such factors as the instantaneous motion of the satellite, the canted geomagnetic dipole and the orbital eccentricity. Furthermore, an optimum design is sought to obtain a control pattern to maximize the effective torque, as well as to minimize the transverse torque.

Equations of Motion

Three major coordinate frames are introduced. The inertial frame Φ_I shown in Fig. 1 is a geocentric coordinate system, the unit vector $\mathbf{k}_I$ being along the polar axis and $\mathbf{i}_I$ along the vernal line. The orbital frame Φ_O is fixed with respect to the orbit, $\mathbf{k}_O$ being along the orbit normal line and $\mathbf{i}_O$ along the ascending node. The body frame Φ_B shown in Fig. 2 is fixed with respect to the satellite body, with $\mathbf{k}_B$ lying in the direction of the spin axis. The Euler angles ψ, θ, and ϕ determine the attitude with respect to the orbital plane, as shown in Fig. 2.

Rigid body dynamics is described such that

$$d\mathbf{H}/dt = \mathbf{T} + \mathbf{D} \tag{1}$$

where $\mathbf{H}$ represents the total angular momentum of the satellite, $\mathbf{T}$ the control torque, and $\mathbf{D}$ the disturbing torque. $\mathbf{T}$ is given by the vector product of the magnetic dipole moment $\mathbf{M}$ and the local geomagnetic induction $\mathbf{B}$, namely, $\mathbf{T} = \mathbf{M} \times \mathbf{B}$. To control the direction of the spin axis, the magnetic dipole should be colinear with the spin axis $\mathbf{k}_B$, and, to control the spin rate, the dipole should be normal to the spin axis. Thus

$$\begin{aligned} \mathbf{M} &= U\mathbf{k}_B \quad \text{for spin axis control} \\ &= V\mathbf{i}_B \quad \text{for spin rate control} \end{aligned} \tag{2}$$

The body-fixed version of Eq. (1) can be written for the $\mathbf{i}_B$, $\mathbf{j}_B$, and $\mathbf{k}_B$ axes, respectively, as

$$\begin{aligned} &I_p(d\omega_1/dt) - (I_p - I_s)\omega_2\omega_3 = T_{B1} \\ &I_p(d\omega_2/dt) - (I_s - I_p)\omega_1\omega_3 = T_{B2} \\ &I_s(d\omega_3/dt) \qquad\qquad\quad\;\; = T_{B3} \end{aligned} \tag{3}$$

Angular rate of the satellite $\boldsymbol{\omega}$ and $\mathbf{T}$ are decomposed in the body frame as $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)$ and $\mathbf{T} = (T_{B1}, T_{B2}, T_{B3})$. The orbital frame version of Eq. (1) can be derived by substituting Euler angles into Eq. (3) as

$$\begin{aligned} &-I_p \sin\theta \cdot \ddot{\psi} - 2I_p\dot{\psi}\dot{\theta}\cos\theta + I_s(\dot{\psi}\cos\theta + \dot{\phi}) \cdot \dot{\theta} \\ &\qquad = T_{B1}\cos\phi - T_{B2}\sin\phi = T_{S1} \\ &I_p\ddot{\theta} + [I_s\dot{\phi} + (I_s - I_p)\cos\theta \cdot \dot{\psi}]\sin\theta \cdot \dot{\psi} \\ &\qquad = T_{B1}\sin\phi + T_{B2}\cos\phi = T_{S2} \\ &I_s(d/dt)(\dot{\phi} + \dot{\psi}\cos\theta) = T_{B3} = T_{S3} \end{aligned} \tag{4}$$

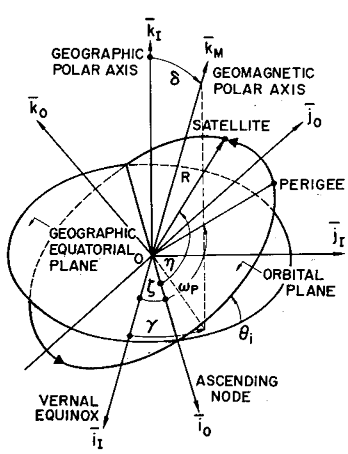

Fig. 1 Inertial coordinate frame Φ_I and orbital coordinate frame Φ_O.

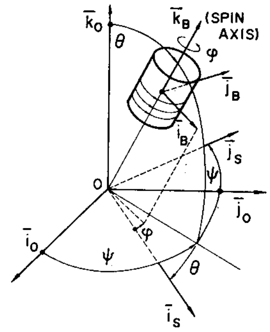

Fig. 2 Body coordinate frame Φ_B, and relation between Φ_O and Φ_B.

where (T_{S1}, T_{S2}, T_{S3}) is defined as the torque components around the $\mathbf{i}_S$, $\mathbf{j}_S$, $\mathbf{k}_S$ axes, as shown in Fig. 2.

Control torque can be expressed for the spin axis control as

$$\begin{aligned} T_{S1} &= U(\sin\psi B_{O1} - \cos\psi B_{O2}) \\ T_{S2} &= U(\cos\psi\cos\theta B_{O1} + \sin\psi\cos\theta B_{O2} - \sin\theta B_{O3}) \\ T_{S3} &= 0 \end{aligned}$$

where (B_{O1}, B_{O2}, B_{O3}) is the components of the geomagnetic induction $\mathbf{B}$ in Φ_O. It is necessary to express these components as a function of the satellite position η. Description in detail to yield the geomagnetic components can be found in Ref. 7, which gives the following results for a centered dipole model:

$$\begin{aligned} B_{O1}/D_m = {} & (\tfrac{3}{2})\sin\theta_i \sin2\eta\cos\delta + \\ & (\tfrac{1}{2})\{\cos(\gamma - \zeta) + \tfrac{3}{2}[(1 + \cos\theta_i)\cos(2\eta - \gamma + \zeta) + \\ & (1 - \cos\theta_i)\cos(2\eta + \gamma - \zeta)]\}\sin\delta \end{aligned} \tag{5a}$$

$$\begin{aligned} B_{O2}/D_m = {} & (\tfrac{1}{2})\sin\theta_i(1 - 3\cos2\eta)\cos\delta + \\ & (\tfrac{1}{2})\{\cos\theta_i \sin(\gamma - \zeta) + \\ & (\tfrac{3}{2})[(1 + \cos\theta_i)\sin(2\eta - \gamma + \zeta) + \\ & (1 - \cos\theta_i)\sin(2\eta + \gamma - \zeta)]\}\sin\delta \end{aligned} \tag{5b}$$

$$B_{O3}/D_m = -\cos\theta_i\cos\delta + \sin\theta_i\sin(\gamma - \zeta) \cdot \sin\delta \tag{5c}$$

where D_m represents geomagnetic dipole moment and δ geomagnetic polar axis declination. An approximate expression for a simplified Earth polar dipole ($\delta = 0$) becomes

$$\begin{pmatrix} B_{O1} \\ B_{O2} \\ B_{O3} \end{pmatrix} = D_m \begin{pmatrix} \tfrac{3}{2}\sin\theta_i\sin2\eta \\ \tfrac{1}{2}\sin\theta_i(1 - 3\cos2\eta) \\ -\cos\theta_i \end{pmatrix} \tag{6}$$

If we use Eq. (6), we finally obtain

$$\begin{aligned} T_{S1} &= (-UD_m/2)\sin\theta_i[\cos\psi - 3\cos(2\eta - \psi)] \\ T_{S2} &= (UD_m/2)\{\sin\theta_i\cos\theta[\sin\psi + 3\sin(2\eta - \psi)] + \\ &\qquad \cos\theta_i\sin\theta\} \end{aligned} \tag{7}$$

Control Law Generation

Now, let us consider the derivation of a suitable control law to govern the polarity of the dipole moment. To control the direction of the spin axis, the desired state $\mathbf{H}_f$, in terms of angular momentum, can be expressed as $\mathbf{H}_f = I_s\Omega\mathbf{k}_f$, where $\mathbf{k}_f$ represents the desired direction of the spin axis. On the other hand, the present state is $\mathbf{H} = I_p\omega_1\mathbf{i}_B + I_p\omega_2\mathbf{j}_B + I_s\omega_3\mathbf{k}_B$. The difference between $\mathbf{H}_f$ and $\mathbf{H}$ is considered as the error vector, $\mathbf{E} = \mathbf{H}_f - \mathbf{H}$. The objective is to reduce $\mathbf{E}$ to zero.

Substituting this relation into Eq. (1), we obtain

$$\begin{aligned} (d\mathbf{E}/dt) &= -\mathbf{T} - \mathbf{D} = -U(\mathbf{k}_B \times \mathbf{B}) - \mathbf{D} \\ (dE^2/2dt) &= -U\mathbf{E} \cdot (\mathbf{k}_B \times \mathbf{B}) - \mathbf{E} \cdot \mathbf{D} \end{aligned}$$

The sufficient asymptotic stability condition for $\mathbf{E}$ is $dE^2/dt \le 0$. Thus this yields the control criteria $U\mathbf{E} \cdot (\mathbf{k}_B \times \mathbf{B}) \ge 0$ under the assumption that $\mathbf{D}$ is negligibly small as compared to $\mathbf{T}$. Let us assume U to act in the bang-bang manner and define the switching function S as

$$S \equiv \mathbf{E} \cdot (\mathbf{k}_B \times \mathbf{B}) \tag{8}$$

then the control criteria to govern the polarity of U are expressed as

$$\begin{aligned} U &= \alpha^2 \quad \text{when } S > 0 \\ U &= -\alpha^2 \quad \text{when } S < 0 \end{aligned} \tag{9}$$

If the polarity of the dipole moment is selected according to the sign of the switching function, the magnitude of error always decreases. Therefore, the desired orientation can eventually be achieved from any initial state.

To control the spin rate, a similar control law is deduced from the same analysis as

$$\begin{aligned} V &= \beta^2 \quad \text{when } S > 0 \\ V &= -\beta^2 \quad \text{when } S < 0 \end{aligned} \tag{10}$$

S is also the switching function to govern the polarity of the dipole moment V

$$S \equiv \mathbf{E} \cdot (\mathbf{i}_B \times \mathbf{B}) \tag{11}$$

where $\mathbf{E} = \mathbf{H}_f - \mathbf{H}$, $\mathbf{H}_f = I_s\Omega_f\mathbf{k}_B$ and Ω_f is the desired spin rate.

The net torque vector produced does not always coincide with the required torque vector. The effective torque T_{eff}, which is defined as the torque to reduce the error, can be expressed as the scalar product of the net torque $\mathbf{T}$ and the unit error vector $\mathbf{E}/E$, such that

$$T_{\text{eff}} = (\mathbf{E}/E) \cdot \mathbf{T} = (U/E)\mathbf{E} \cdot (\mathbf{k}_B \times \mathbf{B}) = (U/E)S \tag{12}$$

The effective torque is proportional to the proposed switching function. This implies that the applied dipole, if its polarity is selected in accordance with Eq. (9), produces, at every instant, a positive effective torque to reduce the error. This demonstrates the validity of the above derived control function.

Formulation of the switching function for the spin axis orientation from (θ,ψ) to (θ_f,ψ_f) is as follows; $\mathbf{k}_B = (\cos\psi \times \sin\theta,\ \sin\psi \sin\theta,\ \cos\theta)$ and $\mathbf{k}_f = (\cos\psi_f \sin\theta_f,\ \sin\psi_f \sin\theta_f,\ \cos\theta_f)$ in Φ_O frame. Then,

$$\mathbf{k}_B \times \mathbf{B} = \begin{pmatrix} \sin\psi \sin\theta \cdot B_{O3} - \cos\theta \cdot B_{O2} \\ \cos\theta \cdot B_{O1} - \cos\psi \sin\theta \cdot B_{O3} \\ \cos\psi \sin\theta \cdot B_{O2} - \sin\psi \sin\theta \cdot B_{O1} \end{pmatrix} \begin{pmatrix} \mathbf{i}_O \\ \mathbf{j}_O \\ \mathbf{k}_O \end{pmatrix} \tag{13}$$

And the error $\mathbf{E}$ can be decomposed in the Φ_O frame as

$$\mathbf{E} = I_s\Omega\mathbf{k}_f - (I_p\omega_1\mathbf{i}_B + I_p\omega_2\mathbf{j}_B + I_s\omega_3\mathbf{k}_B)$$

$$= I_s \begin{pmatrix} \Omega \cos\psi_f \sin\theta_f - \dot\phi \cos\psi \sin\theta - \\ \qquad \dot\psi \cos\psi \cos\theta \sin\theta \\ \Omega \sin\psi_f \sin\theta_f - \dot\phi \sin\psi \sin\theta - \\ \qquad \dot\psi \sin\psi \cos\theta \sin\theta \\ \Omega \cos\theta_f - \dot\phi \cos\theta - \dot\psi \cos^2\theta \end{pmatrix} \begin{pmatrix} \mathbf{i}_O \\ \\ \mathbf{j}_O \\ \\ \mathbf{k}_O \end{pmatrix} +$$

$$I_p \begin{pmatrix} \dot\theta \sin\psi + \dot\psi \cos\psi \cos\theta \sin\theta \\ -\dot\theta \cos\psi + \dot\psi \sin\psi \cos\theta \sin\theta \\ -\dot\psi \sin^2\theta \end{pmatrix} \begin{pmatrix} \mathbf{i}_O \\ \mathbf{j}_O \\ \mathbf{k}_O \end{pmatrix} \tag{14}$$

Such simplifications as $|\dot\psi \sin\theta|\ |\dot\theta| \ll \Omega$ and $\dot\phi + \dot\psi \times \cos\theta = \Omega$ render the essential aspect of the switching function easy to understand. Then, $\mathbf{E}$ and S become

$$\mathbf{E} = I_s\Omega \begin{pmatrix} \cos\psi_f \sin\theta_f - \cos\psi \sin\theta \\ \sin\psi_f \sin\theta_f - \sin\psi \sin\theta \\ \cos\theta_f - \cos\theta \end{pmatrix} \begin{pmatrix} \mathbf{i}_O \\ \mathbf{j}_O \\ \mathbf{k}_O \end{pmatrix} \tag{15}$$

$$\begin{aligned} S/I_s\Omega = {} & (\sin\psi_f \sin\theta_f \cos\theta - \cos\theta_f \sin\psi \sin\theta)B_{O1} + \\ & (\cos\theta_f \cos\psi \sin\theta - \cos\psi_f \sin\theta_f \cos\theta)B_{O2} + \\ & (\cos\psi_f \sin\theta_f \sin\psi \sin\theta - \sin\psi_f \sin\theta_f \cos\psi \sin\theta)B_{O3} \end{aligned} \tag{16}$$

Substituting Eq. (5) into Eq. (16), we obtain the final form of the switching function.

Analysis for a Simplified Model

First let us consider the control governed by the derived switching function, assuming, for a basic analysis and comparison study, such a simplified model as the Earth polar dipole ($\delta = 0$) and the circular orbit ($e = 0$). Then S becomes

$$\begin{aligned} S = D_m'\{ & \tfrac{3}{2}\sin\theta_i[\sin\theta_f \cos\theta \cos(2\eta - \psi_f) - \\ & \cos\theta_f \sin\theta \cos(2\eta - \psi) + \tfrac{1}{3}(\cos\theta_f \cos\psi \sin\theta - \\ & \cos\psi_f \sin\theta_f \cos\theta)] + \cos\theta_i \sin(\psi_f - \psi) \sin\theta_f \sin\theta\} \end{aligned} \tag{17}$$

where $D_m' \equiv I_s\Omega D_m$

Elevation Angle Control

Elevation angle control is defined as the control to decrease the error in the plane normal to the orbital plane. Initially, the azimuth angle ψ is aligned to ψ_f and only the control of elevation angle (θ to θ_f) is required. For analytical development, it can be assumed that ψ is sensibly invariant during the operation. Then, Eq. (17) becomes

$$\begin{aligned} S/I_s\Omega &= \sin(\theta_f - \theta)(B_{O1} \sin \psi - B_{O2} \cos \psi) \\ &= \tfrac{3}{2}\sin \theta_i \sin(\theta_f - \theta)[\cos(2\eta - \psi) - \tfrac{1}{3}\cos \psi] \cdot D_m \end{aligned} \tag{18}$$

For a particular mission where the desired spin axis is normal to the orbital plane (wheel type stabilization mode), $\mathbf{k}_f$ is equal to $\mathbf{k}_O$, and θ_f is equal to zero. Then, we have

$$S/I_s\Omega = \tfrac{3}{2}\sin\theta_i \sin\theta[\tfrac{1}{3}\cos\psi - \cos(2\eta - \psi)] \cdot D_m \tag{19}$$

As has been reported,[1-3] polarity reversal four times every orbit is required. However, the switching points differ from those that have been reported.

Azimuth Angle Control

Another mission is selected where only the azimuth angle ψ is controlled to the angle ψ_f. For this case, we have, from Eq. (17)

$$\begin{aligned} S/I_s\Omega = \{ & \tfrac{3}{2}\sin\theta_i \cos\theta \sin\theta[\cos(2\eta - \psi_f) - \cos(2\eta - \psi) - \\ & \tfrac{1}{3}(\cos\psi_f - \cos\psi)] + \cos\theta_i \sin^2\theta \sin(\psi_f - \psi)\} \cdot D_m \end{aligned} \tag{20}$$

and, for a mission wherein the spin axis is aligned into the orbital plane ($\theta = \pi/2$), we obtain

$$S/I_s\Omega = -\sin(\psi_f - \psi)B_{O3} = \cos\theta_i \sin(\psi_f - \psi) \cdot D_m \tag{21}$$

No switching is required for a ψ control in the orbital plane.

Controlling the spin axis from (θ, ψ) to (θ_f, ψ_f) can be attained by rotation around the axis parallel to the $(\mathbf{i}_O, \mathbf{j}_O)$ plane (θ-control) and around the $\mathbf{k}_O$ axis (ψ-control for $\theta = \pi/2$). Therefore, the switching function, for any desired orientation, can be described basically as the combination of Eqs. (18) and (21).

Comparison Study on Control Law

The derived control function is based on the maximum utilization of whatever effective torque is available. On the other hand, the quarter orbit switching[1-3] comes from the principle that the averaged torque would produce the desired control. The switching points obtained from Eq. (18) are illustrated in Fig. 3 in the orbital plane, as compared with the quarter orbit switching line. Both lines are shown differently.

For a θ control, T_{eff} and transverse torque $T_\perp$ are expressed from Eqs. (13) and (15) as

$$\mathbf{T}_{eff} = (\alpha^2 \operatorname{sgn} S)(B_{O1} \sin\psi - B_{O2} \cos\psi) \cdot \mathbf{i}_s = (\alpha^2 \operatorname{sgn} S) B_{eff} \cdot \mathbf{i}_s \quad (22)$$

$$\mathbf{T}_\perp = (\alpha^2 \operatorname{sgn} S)(B_{O1} \cos\psi \cos\theta + B_{O2} \sin\psi \cos\theta - B_{O3} \sin\theta) \cdot \mathbf{j}_s \quad (23)$$

where $B_{eff} \equiv (B_{O1} \sin\psi - B_{O2} \cos\psi)$. $T_\perp$ $(\equiv |\mathbf{T}_\perp|)$ produces a parasitic drift normal to the desired direction. The effective orientation $\Delta\theta$ due to T_{eff}. for a control over a full orbital period, is calculated, from Eq. (22), as

$$I_s\Omega \cdot \Delta\theta = 2\alpha^2\left(-\int_{\eta 1}^{\eta 2} B_{eff} d\eta + \int_{\eta 2}^{\eta 3} B_{eff} d\eta\right)$$

Switching points η_1, η_2, η_3 can be read from Fig. 3 as 35.25°, 144.75°, and 215.25°, respectively, for $\psi = 0$, as an example. Then

$$I_s\Omega \cdot \Delta\theta = 6.34\alpha^2 D_m \sin\theta_i \quad (24)$$

The orientation due to the quarter orbit switching, in which $S_{AV} = \cos(2\eta - \psi)$, becomes[1]

$$I_s\Omega \cdot \Delta\theta_{AV} = \int_0^{2\pi} B_{eff}[\alpha^2 \operatorname{sgn}(S_{AV})] d\eta = 6\alpha^2 D_m \sin\theta_i \quad (25)$$

where the subscript AV represents the averaging method. Comparing Eqs. (24) and (25), we obtain $\Delta\theta/\Delta\theta_{AV} = 1.06$. A 6% increase in control rate is attained, regardless of $(\theta_f - \theta)$ and θ_i, by the proposed function.

The transverse drift is not necessarily reduced to zero by the proposed function; besides, the quarter orbit switching causes no parasitic drift after a full orbital control under the condition of the invariant spin axis.[1] The drift $\Delta\psi$, when $\psi = 0$, as an example, is

$$I_s\Omega \cdot \Delta\psi = \int_0^{2\pi} T_\perp d\eta = 1.35\alpha^2 D_m \cos\theta_i \sin\theta \quad (26)$$

Thus, $\Delta\psi/\Delta\theta = 0.213 \cot\theta_i \sin\theta$. However, for a highly inclined orbit in which a θ control becomes effective, $\Delta\psi$ decreases to zero (for $\theta_i = 75°$, $\Delta\psi/\Delta\theta = 0.056 \sin\theta$) and, for a wheel type stabilization $(\theta \to 0)$, $\Delta\psi$ also reaches zero. Even if $\Delta\psi$ exceeds the allowable level, the derived function will act effectively on both $\Delta\theta$ and $\Delta\psi$ errors. A simulation run in a digital computer confirms the validity of the above results.

Investigation of Control Law—Spin Axis Control

General Control Law

The study now will be extended to a general control law on an elliptical orbit in the rotating canted dipole. An approximate expression for an elliptical orbit is obtained by expanding the orbital equations in the power series with respect to e. Then, $r_1 \doteqdot r[1 - e\cos(\eta - \omega_p)]$ and $\eta_1 \doteqdot \eta + 2e\sin(\eta - \omega_p)$, where the subscript 1 represents the elliptical orbit. Using these relations, the switching points (time) for an elliptical orbit are found to be

$$\eta_{10} \doteqdot \eta_0 - 2e\sin(\eta_0 - \omega_p) \quad (27)$$

where η_0 $(\equiv nt_0)$ is the point for a circular orbit and n the mean orbital motion. The deviation due to e is approximately $-2e\sin(\eta_0 - \omega_p)$, being dependent on η_0 and ω_p. A typical example, when $\omega_p = \pi/2$ and $e = 0.2$, is shown in Fig. 3. The change due to e, the maximum being beyond 20°, is not negligible on a highly eccentric orbit.

Defining $B_i' \equiv B_{Oi}/D_m$ $(i = 1, 2, 3)$ as the uncanted geomagnetic components and B_{1i}' as the canted components, the first-order approximate expression should be $B_{1i}' \doteqdot B_i' + \delta b_i$ $(i = 1, 2, 3)$. From Eq. (5)

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}[\cos\xi + 3(\cos 2\eta \cos\xi + \cos\theta_i \sin 2\eta \sin\xi)] \\ \frac{1}{2}[\cos\theta_i \sin\xi + 3(\sin 2\eta \cos\xi - \cos\theta_i \cos 2\eta \sin\xi)] \\ \sin\theta_i \sin\xi \end{pmatrix} \quad (28)$$

For an elevation control, defining $S' \equiv S/I_s\Omega$ and S_1' for the canted model, we have, from Eq. (18)

$$\begin{aligned} S_1' &\doteqdot S' + \Delta S = S' + \delta \sin(\theta_f - \theta)[b_1 \sin\psi - b_2 \cos\psi] \cdot D_m \\ &= (\tfrac{3}{2}) \sin(\theta_i + \delta \sin\xi) \sin(\theta_f - \theta)[\cos(2\eta - \psi) - \tfrac{1}{3}\cos\psi] \cdot D_m \\ &\quad - (\tfrac{3}{2})(\delta \cos\xi) \sin(\theta_f - \theta)[\sin(2\eta - \psi) - \tfrac{1}{3}\sin\psi] \cdot D_m \quad (29) \end{aligned}$$

The first term shows that $\delta \sin\xi$ results in an equivalent change of the inclination from θ_i to $\theta_i + \delta\sin\xi$, giving no change on the switching point. The second term, on the other hand, causes the switching point deviation as

$$\Delta\eta = -(\delta/2) \cdot (\cos\xi/\sin\theta_i)\{1 - [\sin\psi/3 \sin(2\eta_0 - \psi)]\} \quad (30)$$

which is illustrated in Fig. 4. Moreover, $\Delta\eta$ varies sinusoidally as the canted dipole rotates with a 24-hr period $[\xi = m(t - t_0)]$. For an azimuth control in the orbital plane, a similar procedure leads to Eq. (31).

$$S_1' \doteqdot \cos(\theta_i + \delta \sin\xi) \sin(\psi_f - \psi) \cdot D_m \quad (31)$$

The orbital parameters experience such secular variations as $d\zeta/dt$, $d\omega_p/dt$ and $d\theta_i/dt$. These variations are slow and may be negligible for a control within several orbital periods. It will be advisable, however, for a control over a day to revise the orbital parameters and the geomagnetic components.

The higher terms of the satellite motion, i.e., the nutational motion, is shown quite negligible by a digital simulation, as the period of nutation $(I_s/I_p)\Omega$ is much faster than the orbital motion.

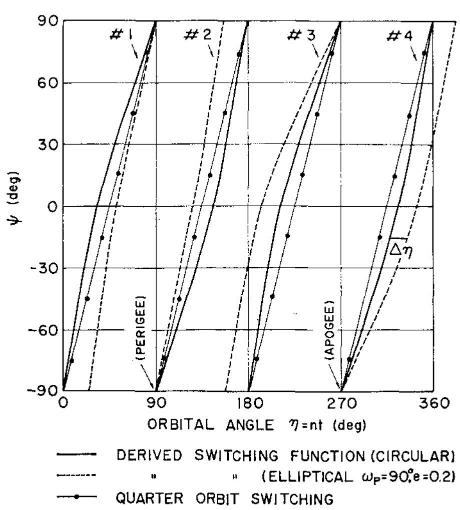

Fig. 3 Comparison of the switching line between the derived switching function and the quarter orbit switching.

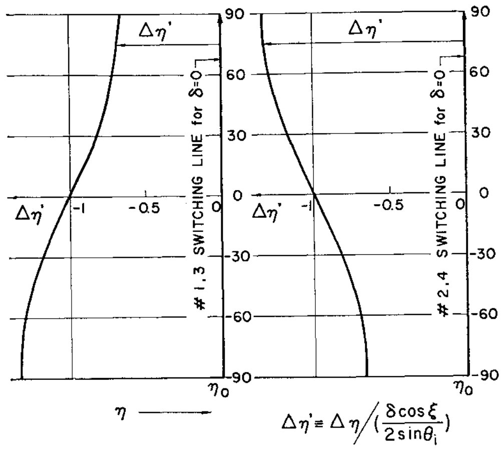

Fig. 4 Switching point deviation due to geomagnetic polar axis declination δ.

Desirable Control Pattern

The preceding analysis has revealed that the switching points are quite sensitive to the azimuth angle of the spin axis, geomagnetic polar axis rotation and orbital eccentricity, and that it may be advisable, for attaining a faster control rate, to select the dipole in accordance with the switching function updated at every instant of operation.

Moreover, it has been found that the magnitude of the effective torque varies almost sinusoidally on the orbit, the maximum value being halfway between the two adjoining switching points and the minimum at the switching point. It can be deduced that a control pattern weighted halfway between the switching points, such as a pulse, triangular, or sine wave, would give faster control than the rectangular pattern. The control rate increment due to the pattern mentioned above is obtained, for the case of e and $\delta = 0$, as shown in Fig. 5, under the condition of the total energy applied being constant. Over 50% increment is achievable by narrowing the pulse width. Performance by the non-negative pulse pattern, designated as the single pulse method, is also examined as an extreme case of the pulse pattern. As the transverse drift cannot be cancelled through every orbital period, the desired point is reached in a spiral mode.

A typical example of performance by the four different control patterns mentioned above is illustrated in Fig. 6 for an elevation control, where the produced torque is calculated assuming the spin axis is sensively invariant during an operation. Thus the actual behavior differs slightly from the figures. The advantage is proved to lie in the narrow pulse pattern, whose location is determined by the derived switching function, if a little more care is taken to reduce the transverse drift.

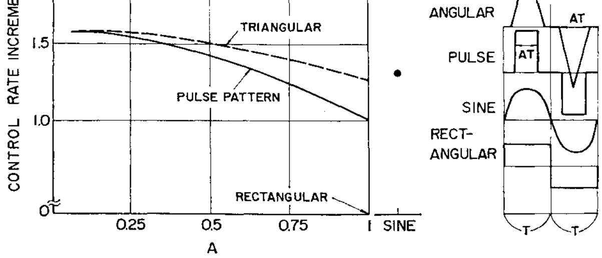

Fig. 5 Control rate increment due to control patterns.

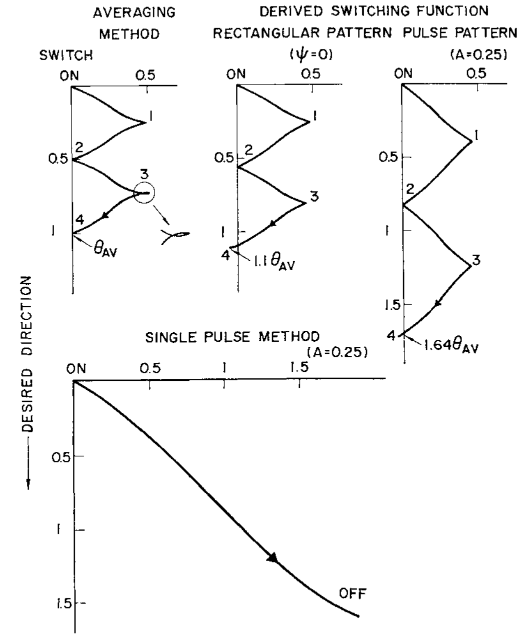

Fig. 6 Typical example of performance by different control laws for one orbital operation ($\delta = e = 0$).

Investigation of Control Law—Spin Rate Control

The desired state for spin rate control can be expressed as $\mathbf{H}_f = I_s\Omega_f\mathbf{k}_B$ where Ω_f is the desired spin rate. Then, the error becomes $\mathbf{E} = \mathbf{H}_f - \mathbf{H} = -I_p\omega_1\mathbf{i}_B - I_p\omega_2\mathbf{j}_B + I_s(\Omega_f - \omega_3)\mathbf{k}_B$. And the switching function is given from Eq. (11) as

$$S = \mathbf{E}\cdot(\mathbf{i}_B \times \mathbf{B}) = I_s(\Omega_f - \omega_3)B_{B2} + I_p\omega_2 B_{B3} \tag{32}$$

The first term corresponds to the effective torque to reduce the spin rate error and the last term to reduce the precession rate ω_2. When there is no initial precession, S becomes $S = I_s(\Omega_f - \omega_3)B_{B2}$. The desirable control pattern is also studied, assuming δ and $e = 0$, on the typical spin rate control models.

$\theta = \psi = \pi/2$ **Case**

From Eq. (6), we obtain

$$B_{B2} = -\cos\phi B_{o1} + \sin\phi B_{o3} \tag{33a}$$

$$= -D_m[\tfrac{3}{2}\sin\theta_i\cos\phi\sin 2\eta + \cos\theta_i\sin\phi] \tag{33b}$$

$$= -D_m(A_1{}^2 + B_1{}^2)^{1/2}\sin(\phi + \phi_{o1}) \tag{33c}$$

where $\phi_{o1} \equiv \tan^{-1}(A_1/B_1) = \tan^{-1}(\tfrac{3}{2}\tan\theta_i\sin 2\eta)$, $A_1 \equiv \tfrac{3}{2}\cdot\sin\theta_i\sin 2\eta$, and $B_1 \equiv \cos\theta_i$. Thus the switching function is found to be

$$S = I_s(\Omega_f - \omega_3)(-D_m)(A_1{}^2 + B_1{}^2)^{\frac{1}{2}}\cdot\sin(\phi + \phi_{o1}) \tag{34}$$

Switching the polarity of the dipole is necessary twice per every spin cycle, and switching has to be made at the angle (phase) $\phi = -\phi_{o1} + n\pi$. The orbital angle η_{max} at which the maximum effective torque is achievable can be obtained by maximizing $(A_1{}^2 + B_1{}^2)$. It yields $|\sin 2\eta| = 1$. Thus $\eta_{max} = \pi/4 \pm n\pi/2$, where the effective torque is described as

$$T_{eff,max} = -\beta^2\,\mathrm{sgn}S \times D_m(1 + \tfrac{5}{4}\sin^2\theta_i)^{\frac{1}{2}}\cdot\sin(\phi + \phi_{o1}) \tag{35}$$

η_{max} is independent of θ_i, but $T_{eff,max}$ becomes larger as the value of θ_i increases.

The dipole moment also produces the torque normal to the effective torque to precess the spin axis as $\mathbf{T} = V(\mathbf{i}_B \times \mathbf{B}) = V(-B_{B3}\mathbf{j}_B + B_{B2}\mathbf{k}_B)$. The first term corresponds to the transverse component $T_\perp$.

$$T_\perp = -VB_{B3} = -V(\cos\psi \sin\theta \cdot B_{O1} + \sin\psi \sin\theta \cdot B_{O2} + \cos\theta \cdot B_{O3}) \quad (36)$$

In case ψ and θ are $\pi/2$, we have $T_\perp = -VD_m \sin\theta_i(1 - 3\cos 2\eta)/2$. Further, in the vicinity of η_{max}, we obtain $T_\perp = -VD_m \sin\theta_i/2$. Representation of $T_\perp$ in a proportional form $(T_\perp/T_{eff})$ is shown in Fig. 7, which indicates that $T_\perp$ becomes minimum in the vicinity of η_{max}. In addition, the effect of $T_\perp$ can be nearly cancelled per one spin cycle as the dipole V changes its polarity twice per spin cycle. Therefore, desirable performance can be obtained if the spin rate control is operated in the vicinity of η_{max}.

$\theta = \pi/2$ and $\psi = 0$ Case

S and T_{eff} are given for the case of $\theta = \pi/2$ and $\psi = 0$ as

$$S = I_s(\Omega_f - \omega_3)(-D_m)(A_2{}^2 + B_2{}^2)^{\frac{1}{2}} \cdot \sin(\phi + \phi_{O2}), \quad T_{eff} = [\beta^2 \operatorname{sgn} S/I_s(\Omega_f - \omega_3)] \cdot S \quad (37)$$

where $A_2 \equiv -\sin\theta_i(1 - 3\cos 2\eta)/2$, $B_2 \equiv \cos\theta_i$ and $\phi_{O2} \equiv \tan^{-1}[\tan\theta_i(3\cos 2\eta - 1)/2]$. The optimum orbital points, the effective torque and the transverse torque are given as

$$\eta_{max} = \pi/2 \pm n\pi \quad (38)$$

$$T_{eff,max} = -\beta^2 \operatorname{sgn} S \times D_m(1 + 3\sin^2\theta_i)^{\frac{1}{2}} \cdot \sin(\phi + \phi_{O2}) \quad (39)$$

$$T_\perp = -\tfrac{3}{2}\beta^2 \operatorname{sgn} S \times D_m \sin\theta_i \sin 2\eta \quad (40)$$

At $\eta = \eta_{max}$, $T_\perp$ is reduced to zero.

$\theta = 0$ Case (Wheel Type)

We have, for $\theta = 0$

$$S = I_s(\Omega_f - \omega_3)(-D_m)\sin\theta_i[(5 - 3\cos 2\eta)/2]^{\frac{1}{2}} \cdot \sin(\psi + \phi + \phi_{O3}) \quad (41)$$

where $\phi_{O3} \equiv \tan^{-1}[(3\cos 2\eta - 1)/(3\sin 2\eta)]$. From Eq. (41) the optimum point should be at $\eta_{max} = \pi/2 \pm n\pi$. At this point $T_{eff,max}$ becomes

$$T_{eff,max} = -2\beta^2 \operatorname{sgn} S \times D_m \sin\theta_i \sin(\psi + \phi + \phi_{O3}) \quad (42)$$

For any other θ and ψ condition, η_{max}, $T_{eff,\,max}$, and $T_\perp$ will be obtained by a similar procedure.

The effective torque becomes maximum at particular points on the orbit, and, in the vicinity of these points, the precession torque in general reaches its minimum level. Thus, it is desirable that the control be performed only around these points.

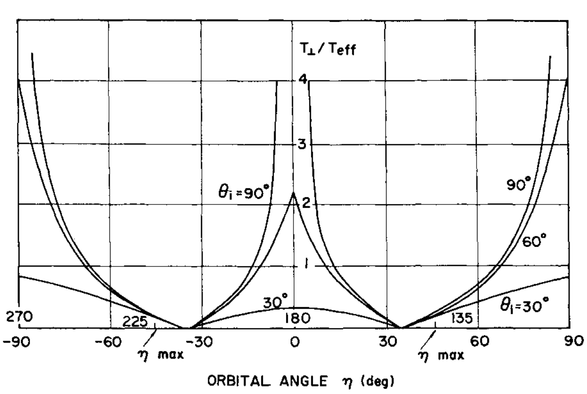

Fig. 7 Ratio of precession torque to effective torque for spin rate control (when $\theta = \psi = \pi/2$).

Control Scheme and Simulation

Control Scheme Mechanization

Here a brief discussion is conducted on how to mechanize the control system. The self-contained onboard control system is, in view of the nature of the proposed function, most advisable. The control system is composed of the control coil and electronic circuits, the attitude sensors and the magnetometers, which enable up-dating the switching function along the orbit. An alternate way would be the programmed control scheme, using the ground station. The control pattern is calculated, in advance, by operating the model of the entire system prepared in a digital computer at the ground. Then, the pattern is commanded to the onboard programmer. Since the prediction of the satellite attitude and the geomagnetic induction is usually possible, this scheme is, in a practical sense, quite applicable.

Simulated Model

The control law derived above will be demonstrated here by digital computer simulation to show its feasibility, assuming the following model.

A model satellite is taken as $I_s = 8.1$ kg-m^2, $I_p = 5.4$ kg-m^2 and $\Omega = 12$ rpm; circular orbit of 1,000 km altitude and θ_i is 30°. The satellite is assumed to be launched from the Tanegashima Island of Japan (E 130° 58′, N 30°22′), injected into orbit over the equator ($\theta_0 = \psi_0 = \pi/2$), and wheel-type stabilization is to be achieved within 2 or 3 days. After initial acquisition, maintaining of spin axis is necessary for preventing attitude drift due to disturbances. Among these, the drift due to the nodal regression is dominant. Therefore, about a 2.7° correction around the $\mathbf{j}_O$ axis ($\Delta\theta = \Delta\Omega_R \sin\theta_i = 2.7°$/day) is necessary every day. The maintaining control is to be exercised within one or two orbits. The amount of decay of spin rate is estimated as from 0.02 to 0.07 rpm per day. The spin rate is to be maintained by about ten minutes activation per day.

From the discussion in the preceding section, a pulse having a fixed width is taken in a practical application as shown in Fig. 8a. Two pulse levels are selected; the high level of 25 ATM^2 is taken for acquisition control and the low level of 5.5 ATM^2 for both acquisition and maintaining.

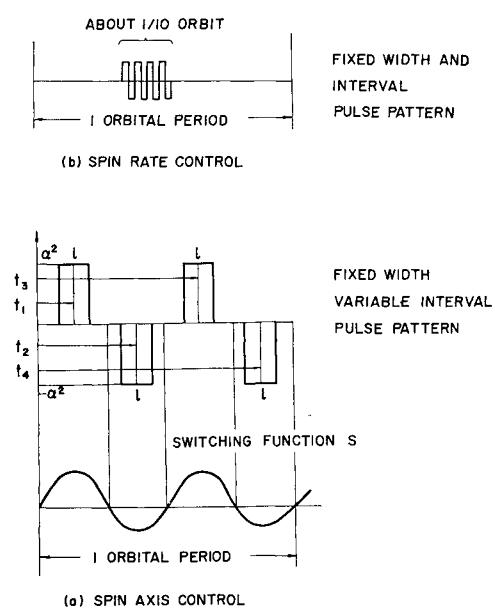

Fig. 8 Proposed control pattern for the on-board dipole moment in a practical application.

Spin rate control is carried out within a limited region in the orbit. The variation of the spin phase to be switched is quite small within this region. Therefore, switching the polarity with the interval fixed is allowable. The pattern is selected as shown in Fig. 8b, the level of the dipole moment being 8.4 ATM^2 and the time activated being about 10 min.

Simulation Results

Figure 9 represents successful performance for the initial acquisition; the initial error of 90° has been corrected within fourteen orbits (about one day), and the low level dipole at

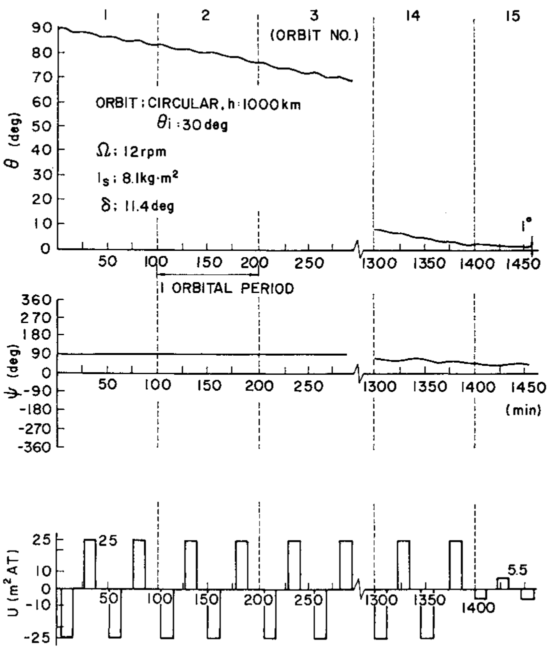

Fig. 9 Initial acquisition of spin axis results, time histories of θ, ψ, U.

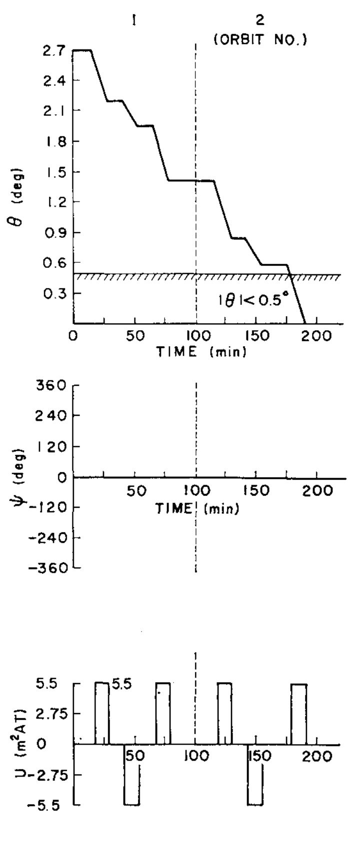

Fig. 10 Spin axis maintaining control results, time histories of θ, ψ, U.

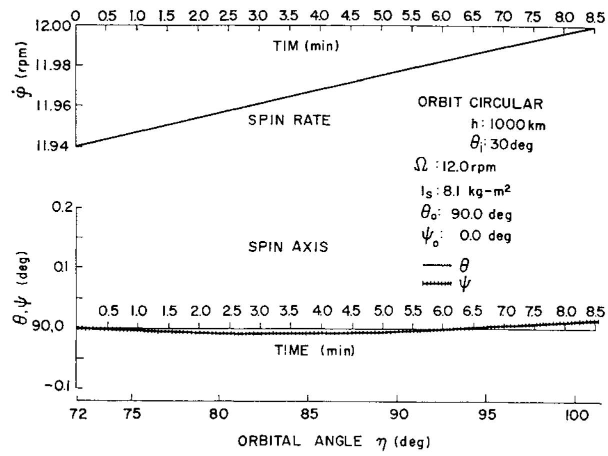

Fig. 11 Spin rate control result.

the final stage has set the spin axis in the desired state within less than 1°. The results for spin axis maintaining shown in Fig. 10 also indicate successful operation; the deviation due to the nodal regression has been corrected within two orbital periods. Moreover, in Fig. 11 almost ideal performance is shown for the spin rate control. The spin rate has reached the desired value after 10 min of operation, with the attitude drift being negligible.

Conclusion

A general control law for geomagnetic attitude stabilization has been developed to govern onboard dipole moment of a satellite. The control law is described in terms of the switching function which has been derived directly from the asymptotic stability condition, and makes possible the maximum utilization of whatever effective torque is available.

For the elevation angle control of the spin axis, switching the polarity of a dipole four times every orbit is required as was presented by the averaging method.[1-3] However, it has been found that the switching point differs from the quarter orbit switching and that this difference makes the spin axis attain the desired orientation faster. The derived function also renders the influence of the canted geomagnetic dipole δ and the orbital eccentricity e easy to understand. The influence of δ is divided into two parts, i.e., $\delta \sin\xi$ and $\delta \cos\xi$; $\delta \sin\xi$ produces an equivalent change in the orbital inclination, and $\delta \cos\xi$ and e causes switching point deviation. The switching point is quite sensitive to the azimuth of the spin axis and the geomagnetic induction, thus it is advisable that the polarity be selected in accordance with the switching function up-dated at every instant of operation.

The narrow pulse pattern, activated halfway between the switching points, has been found desirable in a practical application, since a considerably effective utilization of the applied torque is made possible, and, as a result, the transverse torque is reduced.

The control law for spin rate control has also been developed from the switching function. Limited operation around the selected point on the orbit is desirable, since at these points the effective torque reaches its maximum level and, on the contrary, the torque to cause the spin axis drift is usually at a minimum level.

Simulation run on the typical model satellite has shown the feasibility of the proposed law. The narrow pulse pattern according to the derived law is shown to be quite acceptable. Wheel-type stabilization to maneuver the spin axis through 90° is achieved within a few days; the acuracy is better than 1°, and the time required is reduced by over 50% as compared to that by the quarter orbit pattern. The spin rate is shown to be kept within 0.5% drift by about 10 min activation every day, the pointing of the spin axis being kept undisturbed during the operation.

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